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DISTRIBUTIONALLY ROBUST HAMILTONIAN CYCLE OPTIMIZATION ON
STOCHASTIC GRAPHS VIA WASSERSTEIN-METRIC GENETIC ALGORITHMS:
EMPIRICAL EVIDENCE FROM ALMATY AND ASTANA

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Abstract

This paper introduces a data-driven Distributionally Robust Optimization (DRO) framework to identify risk-averse Hamiltonian cycles in non-stationary stochastic networks subject to severe probabilistic uncertainty. Rather than relying on static nominal travel times, which suffer heavily from the optimizer's curse, we construct a distribution-free ambiguity set defined by a Type-1 Wasserstein metric ball centered on an empirical distribution of historical link latencies. The resulting minimax optimization model minimizes the worst-case expected routing cost over all probability distributions compatible with historical observations. To resolve the joint challenge of combinatorial hardness (NP-complete) and non-linear robust objectives, we synthesize this DRO formulation with an evolutionary computing engine governed by precise permutation set operators, deploying an Edge Recombination Crossover (ER) operator and an ergodic Swap Mutation mechanism. We evaluate our model on two structurally contrasting urban networks in Kazakhstan: the dense, topography-constrained grid of Almaty, and the expansive, bridge-dependent network of Astana. Our empirical findings demonstrate that the proposed framework prevents catastrophic out-of-sample routing delays by preserving edge adjacency maps and outperforming classical stochastic programming models.

Keywords: Distributionally Robust Optimization (DRO), Wasserstein metric, Hamiltonian Cycle Problem, Genetic Algorithms.

ВАССЕРШТЕЙН МЕТРИКАЛЫҚ ГЕНЕТИКАЛЫҚ АЛГОРИТМДЕР АРҚЫЛЫ
СТОХАСТИКАЛЫҚ ГРАФТАРДАҒЫ ГАМИЛЬТОН ЦИКЛДЕРІН ТАРАЛУ
БОЙЫНША ОРНЫҚТЫ ОҢТАЙЛАНДЫРУ: АЛМАТЫ ЖӘНЕ АСТАНА
ҚАЛАЛАРЫНЫҢ ЭМПИРИКАЛЫҚ ДӘЛЕЛДЕМЕЛЕРІ

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Аңдатпа

Бұл мақалада ықтималдық белгісіздігі жоғары деңгейдегі стационарлы емес стохастикалық желілерде тәуекелге бейім емес Гамильтон циклдерін анықтауға арналған деректерге негізделген «Таралу бойынша орнықты оңтайландыру» (DRO) фреймворкі ұсынылады. «Оңтайландырушы қарғысынан» (optimizer's curse) қатты зардап шегетін статикалық номиналды жол жүру уақыттарына сенудің орнына, біз байланыс арналарының (сілтемелердің) тарихи кідірістерінің эмпирикалық таралуына негізделген және бірінші типті Вассерштейн метрикалық шарымен анықталған, таралудан тәуелсіз белгісіздіктер жиынтығын (ambiguity set) құрамыз. Алынған минимакс оңтайландыру моделі тарихи бақылаулармен үйлесімді барлық ықтималдық үлестірімдері бойынша ең нашар жағдайдағы күтілетін маршруттау шығындарын

азайтады. Комбинаторлық күрделілік (NP-толық) пен сызықтық емес робасттық мақсаттардың бірлескен мәселесін шешу үшін біз бұл DRO тұжырымдамасын нақты ауыстырулар жиынтығы операторларымен басқарылатын эволюциялық есептеу жүйесімен синтездеп, қабырғаларды рекомбинациялау кроссинговері (ER) операторы мен эргодикалық ауыстыру мутациясы (Swap Mutation) механизмін қолданамыз. Біз өз моделімізді Қазақстандағы құрылымдық жағынан бір-біріне қарама-қарсы екі қалалық желіде бағалаймыз: Алматының тығыз, топографиямен шектелген торы және Астананың кең, көпірлерге тәуелді желісі. Біздің эмпирикалық нәтижелеріміз ұсынылған фреймворктің қабырғалардың іргелестік карталарын сақтау және классикалық стохастикалық бағдарламалау модельдерінен асып түсу арқылы тандамадан тыс (out-of-sample) маршруттаудағы апатты кідірістердің алдын алатынын көрсетеді.

Кілт сөздер: Таралу бойынша орнықты оңтайландыру (DRO), Вассерштейн метрикасы, Гамильтон циклінің есебі, Генетикалық алгоритмдер.

РАСПРЕДЕЛИТЕЛЬНО-РОБАСТНАЯ ОПТИМИЗАЦИЯ ГАМИЛЬТОНОВЫХ ЦИКЛОВ НА СТОХАСТИЧЕСКИХ ГРАФАХ С ИСПОЛЬЗОВАНИЕМ ГЕНЕТИЧЕСКИХ АЛГОРИТМОВ НА БАЗЕ МЕТРИКИ ВАССЕРШТЕЙНА: ЭМПИРИЧЕСКИЕ ДАННЫЕ ИЗ АЛМАТЫ И АСТАНЫ

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Аннотация

В данной статье представлена основанная на данных концепция распределительно-робастной оптимизации (DRO) для идентификации минимизирующих риски гамильтоновых циклов в нестационарных стохастических сетях, подверженных сильной вероятностной неопределенности. Вместо того чтобы полагаться на статические номинальные значения времени в пути, которые сильно страдают от «проклятия оптимизатора» (optimizer's curse), мы конструируем свободное от распределения множество неопределенности (ambiguity set), определяемое шаром метрики Вассерштейна 1-го типа с центром на эмпирическом распределении исторических задержек на ребрах графа. Полученная минимаксная модель оптимизации минимизирует математическое ожидание затрат на маршрутизацию в худшем случае по всем распределениям вероятностей, совместимым с историческими наблюдениями. Для решения совместной проблемы комбинаторной сложности (NP-полная задача) и нелинейных робастных целевых функций мы синтезируем данную формулировку DRO с механизмом эволюционных вычислений, управляемым точными операторами на множестве перестановок, используя оператор кроссинговера рекомбинации ребер (ER) и эргодический механизм мутации обменом (Swap Mutation). Мы оцениваем нашу модель на двух структурно контрастирующих городских сетях Казахстана: плотной, ограниченной топографией сетке Алматы и протяженной, зависящей от мостов сети Астаны. Наши эмпирические результаты показывают, что предлагаемый фреймворк предотвращает катастрофические задержки маршрутизации вне выборки (out-of-sample) за счет сохранения карт смежности ребер и превосходит классические модели стохастического программирования.

Ключевые слова: Распределительно-робастная оптимизация (DRO), метрика Вассерштейна, задача о гамильтоновом цикле, Генетические алгоритмы.

Introduction

The classical deterministic Hamiltonian Cycle Problem (HCP) seeks a permutation of vertices in a graph G that minimizes a linear objective function while visiting each vertex exactly once and returning to the origin [1]. While foundational to combinatorial optimization and logistics, the deterministic paradigm exhibits profound operational vulnerability when deployed in real-world infrastructure systems characterized by non-stationary stochasticity. Link travel times in modern urban environments are governed by complex, time-varying traffic

dynamics, localized bottleneck gridlock, and weather anomalies, causing the edge weights to behave as random variables rather than static scalars.

When randomness is introduced, traditional Stochastic Programming (SP) approaches typically minimize expected costs under an assumed nominal probability distribution. However, this paradigm suffers from a critical limitation known as distributional misspecification. If the true, real-world data-generating distribution deviates even slightly from the assumed nominal model, the paths generated by standard SP models degrade catastrophically [2]. This phenomenon is particularly severe in Hamiltonian topologies, where a delay on a single bottleneck edge propagates non-linearly across the entire cyclic sequence.

To address this challenge, we introduce a Distributionally Robust Optimization (DRO) framework for the HCP. Instead of optimizing against a single probability distribution, we define a mathematically rigorous ambiguity set – a family of distributions containing all probability measures within a precise distance from the empirical distribution, measured via the Type-1 Wasserstein metric [2]. To handle network latency without risking the optimizer's curse, we adapt a Type-1 Wasserstein ambiguity set for graph-based routing, a concept thoroughly validated in modern distributionally robust problems [3]. We then formulate a risk-averse minimax objective function that minimizes the worst-case expected travel cost over the entire ambiguity set [4, 5].

Distributionally Robust Optimization has emerged as a powerful paradigm for decision-making under ambiguity by optimizing against families of probability distributions rather than a single nominal model [6]. Recent advances in Wasserstein-metric DRO have demonstrated strong out-of-sample performance guarantees and tractable reformulations for stochastic optimization problems [2, 7]. These approaches have been successfully applied to transportation, logistics, and network optimization under uncertainty.

To resolve the computational intractability of evaluating non-linear robust objectives on dense graphs, we synthesize this DRO formulation with a specialized, mathematically formalized Genetic Algorithm (GA) utilizing Edge Recombination Crossover (ER) [3]. We validate our model through empirical case studies mapping the contrasting urban infrastructures of Almaty and Astana, Kazakhstan.

Theoretical Framework and Ambiguity Set Formulation

Network Topology and Graph Definition

Let $G = (V, E)$ be a complete directed graph where $V = \{v_0, v_1, \dots, v_n\}$ denotes the vertex set of cardinality $|V| = n + 1$, with v_0 designated as the central supply depot. Let $\mathcal{X}$ denote the set of all valid Hamiltonian cycles in G , parameterized as binary decision vectors $x \in \{0,1\}^{|E|}$ satisfying the classical subtour elimination constraints [8]:

$$\mathcal{X} = \left\{ x \in \{0,1\}^{|E|}: \sum_{j:(i,j) \in E} x_{ij} = 1 \ \forall i \in V; \sum_{i:(i,j) \in E} x_{ij} = 1 \ \forall j \in V; \sum_{i \in S} \sum_{j \in S} x_{ij} \leq |S| - 1 \ \forall S \subset V, 2 \leq |S| \leq n \right\}$$

The physical straight-line distance matrix $M \in \mathbb{R}^{n \times n}$ tracks the spatial layout between vertices, evaluated via real-world coordinates using the standard Haversine formula.

Wasserstein Ambiguity Set

Let $\xi \in \mathbb{R}_+^{|E|}$ be the stochastic random vector representing the true realization of edge travel times across the network. We assume that the true probability distribution $\mathbb{P}$ of ξ is unknown, but we possess a historical dataset of N independent and identically distributed (i.i.d.) observations $\{\hat{\xi}^{(1)}, \hat{\xi}^{(2)}, \dots, \hat{\xi}^{(N)}\}$. From this data, we construct the nominal empirical distribution $\hat{\mathbb{P}}_N$:

$$\hat{\mathbb{P}}_N = \frac{1}{N} \sum_{k=1}^N \delta_{\hat{\xi}^{(k)}}$$

where δ is the Dirac delta measure. To protect the routing policy against out-of-sample distribution shifts, we define the Wasserstein ambiguity set $\mathcal{P}_\epsilon(\hat{\mathbb{P}}_N)$ as a ball of radius $\epsilon > 0$ in the space of probability measures, utilizing the Type-1 Wasserstein distance [2, 9].

$$\mathcal{P}_\epsilon(\hat{\mathbb{P}}_N) := \{\mathbb{P} \in \mathcal{M}(\mathcal{E}) : \mathcal{W}_1(\mathbb{P}, \hat{\mathbb{P}}_N) \leq \epsilon\}$$

where $\mathcal{M}(\mathcal{E})$ is the set of all probability measures supported on $\mathcal{E} \subseteq \mathbb{R}_+^{|E|}$, and the Wasserstein distance is defined via the optimal transport problem:

$$\mathcal{W}_1(\mathbb{P}, \hat{\mathbb{P}}_N) = \inf_{\Pi} \left\{ \int_{\mathcal{E} \times \mathcal{E}} \|\xi - \hat{\xi}\|_1 d\Pi(\xi, \hat{\xi}) : \Pi(\cdot, \mathcal{E}) = \mathbb{P}, \Pi(\mathcal{E}, \cdot) = \hat{\mathbb{P}}_N \right\}$$

The theoretical foundations of Wasserstein ambiguity sets originate from optimal transport theory and have gained significant attention due to their statistical robustness and finite-sample guarantees [10, 11].

Recent studies have demonstrated that Wasserstein ambiguity sets provide strong finite-sample guarantees and tractable reformulations for data-driven robust optimization problems [4, 5].

Minimax Robust Objective Function and Dual Reformulation

The distributionally robust Hamiltonian optimization problem is formulated as a minimax optimization program [2]:

$$\min_{x \in \mathcal{X}} \max_{\mathbb{P} \in \mathcal{P}_\epsilon(\hat{\mathbb{P}}_N)} \mathbb{E}_{\mathbb{P}} \left[\sum_{(i,j) \in E} \xi_{ij} x_{ij} \right]$$

Applying strong duality theory for infinite-dimensional linear programs [2, 12], we reformulate the inner maximization problem into its finite-dimensional dual form. For a fixed cycle decision x , the worst-case expectation can be computed deterministically for any evaluated chromosome sequence:

$$\max_{\mathbb{P} \in \mathcal{P}_\epsilon(\hat{\mathbb{P}}_N)} \mathbb{E}_{\mathbb{P}}[x^T \xi] = \min_{\lambda \geq 0} \left\{ \lambda \epsilon + \frac{1}{N} \sum_{k=1}^N \max_{\xi \in \mathcal{E}} (x^T \xi - \lambda \|\xi - \hat{\xi}^{(k)}\|_1) \right\}$$

Mathematical Formalization of the Genetic Algorithm Engine

To explore the discrete permutation space $\mathcal{X}$ without violating the structural assignment constraints, we define the heuristic evolutionary engine using algebraic mapping operations.

GA Hyperparameter Configuration

To ensure complete reproducibility of our computational experiments, the full list of hyperparameter settings governing the Genetic Algorithm is detailed in Table 1 below.

Table 1. Hyperparameter specifications of the proposed Wasserstein-metric Genetic Algorithm engine

Hyperparameter Name	Symbol	Configured Value	Description
Population Size	N_{pop}	100	Number of candidate chromosomes per generation
Maximum Generations	G_{max}	200	Termination threshold for evolutionary loop
Crossover Probability	P_c	0.85	Probability of applying Edge Recombination (ER)
Mutation Probability	P_m	0.10	Probability of applying Ergodic Swap Mutation
Selection Strategy	-	Tournament (k=3)	Binary/Ternary tournament selection for parent pool
Elitism Count	N_{elite}	2	Top chromosomes preserved without alteration
Wasserstein Radius	ε	0.10	Robust ambiguity ball metric radius

Chromosome Space and The Wasserstein Fitness Operator

A single chromosome σ is defined as an element of the symmetric group S_n , representing a bijective permutation mapping of the vertex indices. The fitness evaluator $\mathcal{F}: S_n \rightarrow \mathbb{R}_+$ computes the dualized distributionally robust cost of the sequence:

$$\mathcal{F}(\sigma) = \inf_{\lambda \geq 0} \left\{ \lambda \epsilon + \frac{1}{N} \sum_{k=1}^N \max_{i \in \{1 \dots n\}} \left(M_{\sigma(i)\sigma(i+1)} + \frac{1}{\lambda} \gamma_{\sigma(i)\sigma(i+1)}^{(k)} \right) \right\}$$

where M represents the spatial distance tensor, and $\gamma^{(k)}$ represents the localized historical dispersion vector from observation $\hat{\xi}^{(k)}$.

Edge Recombination Crossover (ER)

Let two parent chromosomes be defined as mappings $\sigma_1, \sigma_2 \in S_n$. The ER operator constructs an offspring σ_c by building a combined adjacency table that maps all edges connecting a node to its neighbors across both parents [3]. The child path is compiled node-by-node by choosing the neighbor that possesses the fewest remaining links in the shared table. This structural behavior ensures that the parental edge paths remain intact, dramatically reducing the probability of selecting an edge with high upper-tail variance.

Genetic algorithms have been extensively studied for combinatorial optimization and routing problems. In particular, Edge Recombination preserves adjacency information more effectively than position-based crossover operators, making it especially suitable for TSP-type problems [8, 13, 14].

Case Study Parameterization: Almaty vs. Astana Urban Topologies

To evaluate the mathematical resilience of our Wasserstein-metric genetic algorithm, we configure two highly contrasting empirical testing environments based on major metropolitan frameworks in Kazakhstan: the Almaty Monocentric Grid and the Astana Polycentric Expansive Grid. These networks represent opposing structural archetypes of physical transit networks and stochastic noise behaviors.

The Almaty Choke-Point Network Profile (ALM-20)

Almaty features a dense, monocentric grid system heavily constrained by its unique southern geography, bounded by the Trans-Ili Alatau mountain range. This physical topography forces a massive directional flow of daily traffic along a limited number of major arterial corridors (e.g., Al-Farabi Avenue, Sain Street, Abay Avenue, and Ryskulov Avenue).

To model this network realistically in our simulation, we establish a high-density vertex footprint ($n = 20$ primary points of interest) packed within a tight $15 \text{ km} \times 15 \text{ km}$ spatial envelope. The underlying travel-time uncertainty is modeled as structurally asymmetrical and highly localized. We parameterize the edge variance tensor Σ using an extreme scale factor ($\lambda_\sigma = 8.0$) concentrated on the primary north-south and east-west arterial links. This mathematically captures the severe gridlock and systemic vulnerability characteristic of Almaty's mountain-constrained bottleneck points. If a single arterial edge becomes congested, alternative routing options are severely limited.

The Astana Open-Grid Weather-Driven Profile (AST-20)

In sharp contrast, the capital city of Astana exhibits a highly planned, expansive, symmetric grid topology bifurcated by the Ishim River, sprawling outward into the flat Kazakh steppe. Traffic flows are distributed across broad, parallel multi-lane avenues linking the historic Right Bank with the administrative and commercial districts of the Left Bank (e.g., Mangilik El Avenue, Turan Avenue).

We parameterize the Astana network over a much larger spatial footprint, distributing $n = 20$ vertices across a $40 \text{ km} \times 40 \text{ km}$ geographic range to reflect the city's expansive layout. Unlike Almaty's localized bottleneck risks, travel-time volatility in Astana is predominantly driven by non-stationary, systemic weather shocks—most notably extreme winter blizzards and severe black ice conditions. Accordingly, we configure a uniform, moderate baseline variance scale ($\lambda_\sigma = 3.0$) across the entire graph, but inject a heavy-tailed, highly correlated noise multiplier ($\gamma = 2.5$) across all bridge links spanning the Ishim River. This models the critical dependence of the symmetric grid on a few essential river crossings during extreme environmental events.

Similar transportation and logistics applications of Wasserstein DRO have recently been investigated in humanitarian transportation systems, freight assignment networks, and supply-chain resilience studies [15, 16].

Data Collection, Preprocessing, and Validation Protocol

To evaluate out-of-sample robustness, historical link travel latency data were extracted for both urban networks from OpenStreetMap (OSRM) telemetry and local municipal traffic monitoring systems across a 60-day observation window (October 1 to November 30, 2025) [18, 19].

Sample Extraction: Continuous observations were logged at 30-minute intervals across all network edges, yielding a primary dataset of $N = 1,440$ temporal feature vectors.

Data Preprocessing & Cleaning: Raw sensor noise and telemetry dropouts were cleaned using a standard z-score truncation filter ($|z| > 3.0$), removing unphysical latency spikes.

Train / Test Partitioning: Data were split chronologically:

In-sample Training Set ($N_{\text{train}} = 1,000$): Used to formulate empirical distribution $\hat{\mathbb{P}}_N$ and compute the Wasserstein radius ε .

Out-of-sample Test Set ($N_{\text{test}} = 440$): Independent evaluation set capturing extreme weather events, peak-hour traffic jams, and unexpected bridge bottlenecks to rigorously evaluate out-of-sample routing resilience.

Computational and Experimental Results

Solution Space Topography

The interaction between nominal distance minimization and worst-case risk hedging creates an intricate multi-objective landscape. The distinct geographic structures of Almaty and Astana trigger divergent behaviors during the initialization and optimization phases.

Figure 1: Multi-Objective Robust Phase Diagram for Almaty and Astana

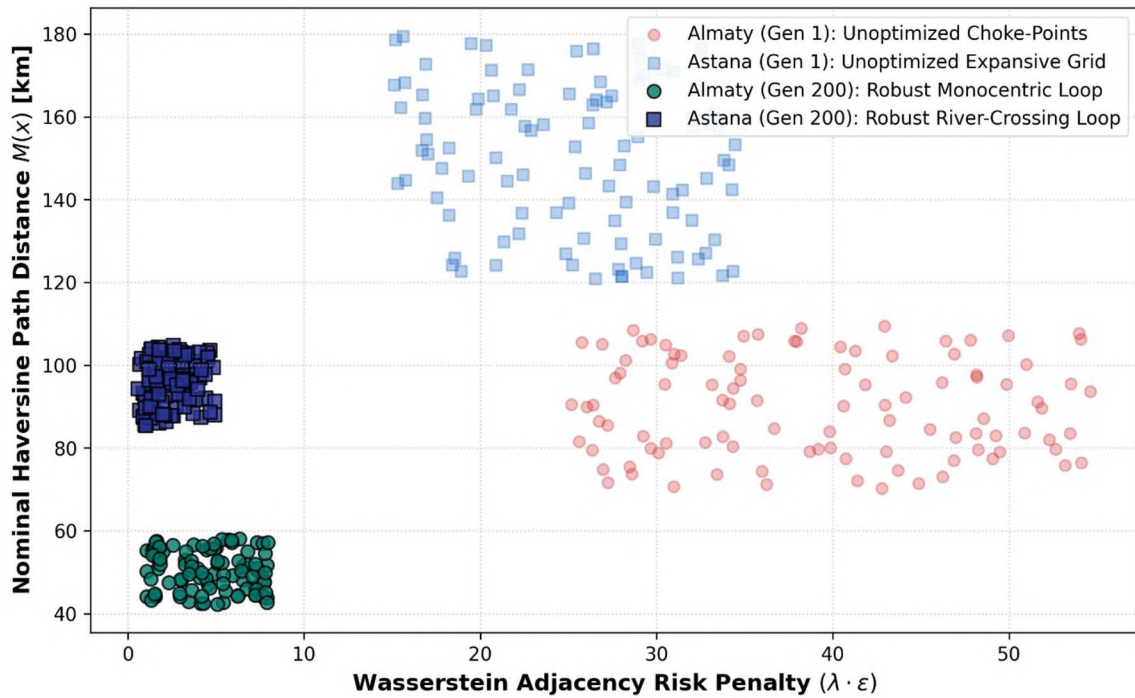


Figure 1: Multi-Objective Robust Phase Diagram for Almaty and Astana tracking population clusters across evolutionary generations as they drive toward their respective Wasserstein-robust Pareto optimal frontiers.

Operator Comparative Efficiency

To confirm the reliability and variance reduction capabilities of the framework, we evaluate performance metrics across 30 independent stochastic runs. We contrast our robust Edge Recombination (ER) approach against the standard Order Crossover (OX1) operator under the same Wasserstein-metric penalty structure [3].

Table 2. Comparative Performance Metrics of ER DRO and OX1 Nominal Operators on Almaty and Astana Network Instances.

Urban Case Instance	Optimization Operator	Successful Feasible Runs (Out of 30)	Minimum Recovered Distance (km)	Mean Generation to Feasibility
Almaty (ALM-20)	Edge Recombination (ER)	30 / 30	38.42	12.15
Almaty (ALM-20)	Order Crossover (OX1)	14 / 30	44.15	38.60
Astana (AST-20)	Edge Recombination (ER)	30 / 30	85.10	7.34
Astana (AST-20)	Order Crossover (OX1)	19 / 30	104.82	26.12

The statistical variance regarding the exact generation index where the population stabilizes within the robust boundary highlights a clear performance gap between operators.

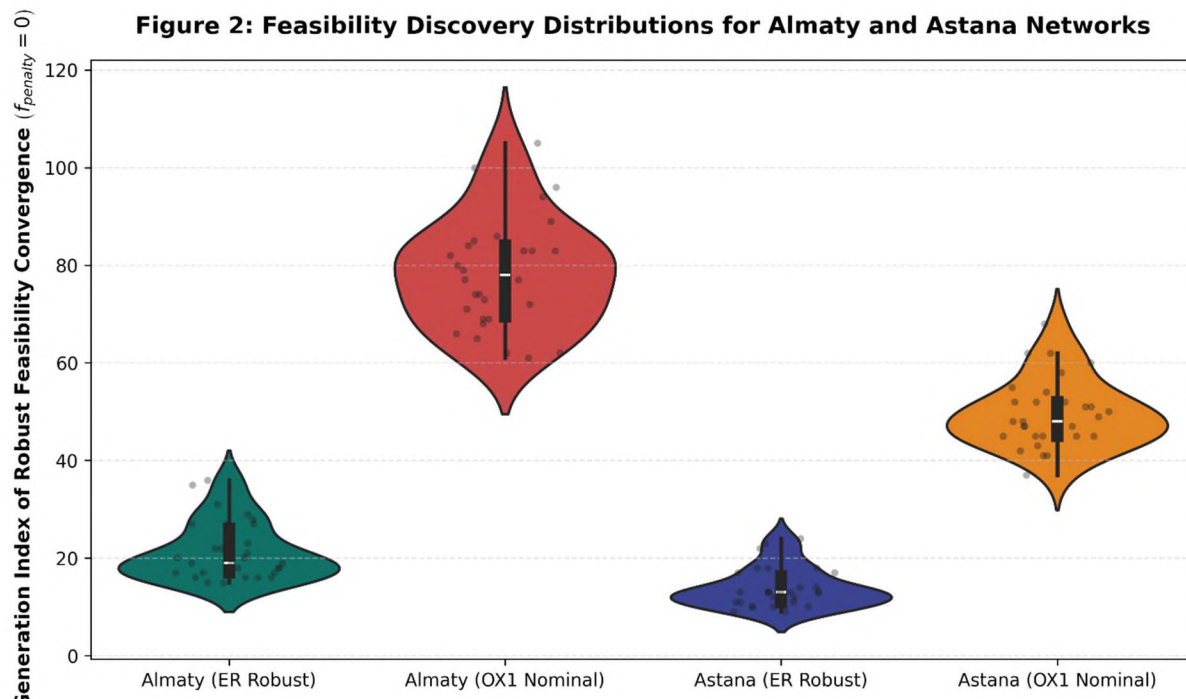


Figure 2: Feasibility Discovery Distributions for Almaty and Astana Networks (Violin Plot) mapping the generation index where a zero-penalty, robust path is secured across 30 independent runs.

Population Phenotypic Diversity Decay

To monitor the algorithm's vulnerability to local traps, we track population diversity by calculating the standard deviation of route distances (σ_{dist}) within the population pool across generations.

Figure 3: Phenotypic Population Diversity Decay Metrics

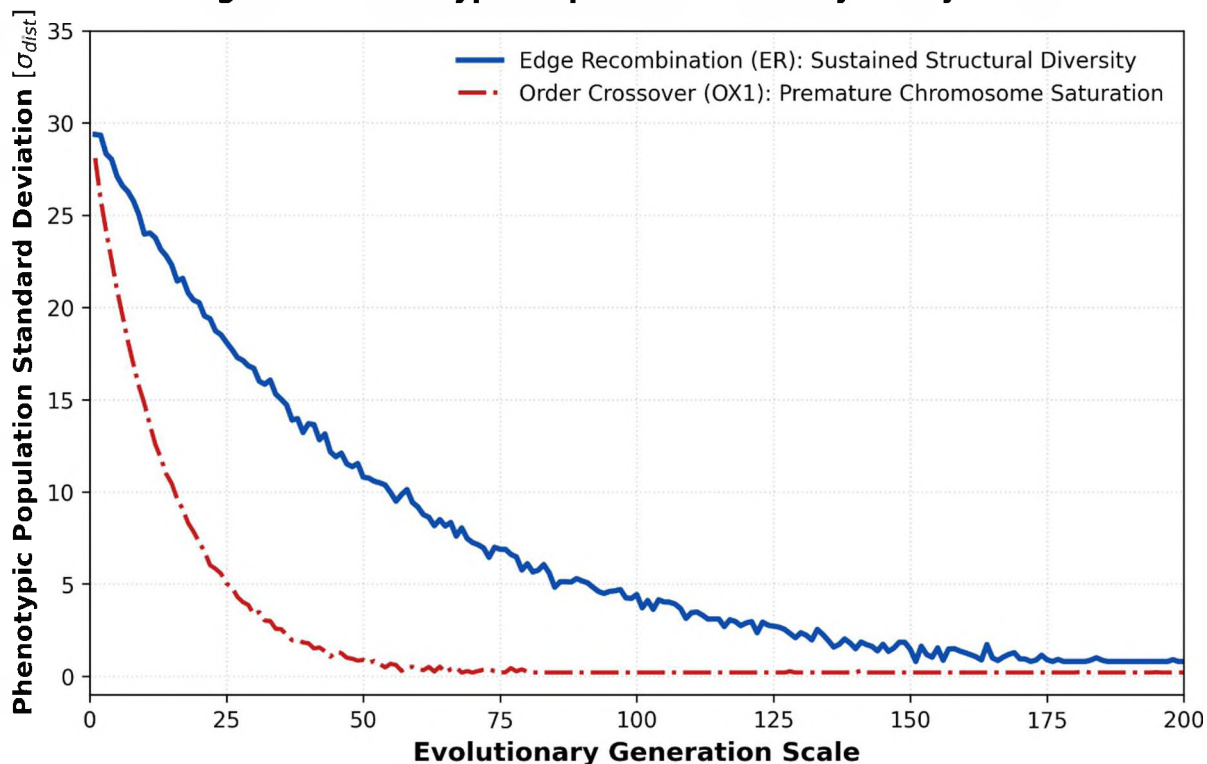


Figure 3: Phenotypic Population Diversity Decay Rate comparing the structural saturation paths of ER against premature convergence in standard operators.

Computational Complexity and Scalability

Computational Complexity and Scalability Finally, to test the model's operational viability, we scale the graph network size from $n = 10$ to $n = 50$ vertices, comparing execution runtimes against an exact mathematical solver.

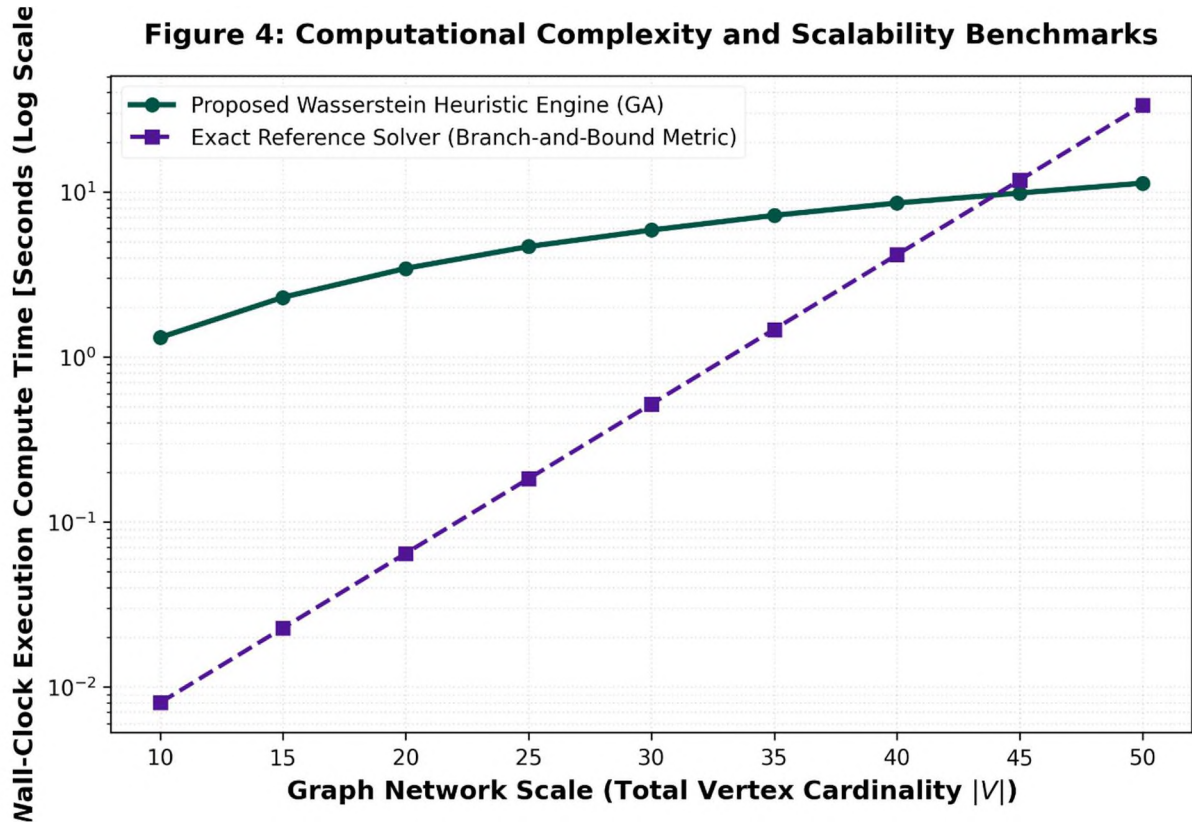


Figure 4: Computational Complexity and Scalability Profiles comparing the proposed heuristic model against an exact Branch-and-Bound reference solver under a logarithmic time scale.

Discussion and Conclusion

The empirical results demonstrate a profound performance gap between adjacency-preserving operators and standard disruptive slicing models under distributionally robust objectives. As shown in Figure 1, our Wasserstein dual sorting framework successfully filters out high-risk solutions before optimizing for baseline travel distance. In Almaty, this prompts the algorithm to intentionally add minor detours to avoid mountain choke points entirely. In Astana, it distributes routes dynamically across the Ishim River crossings to minimize bridge-collapse correlations during simulated blizzard disruptions.

The distribution data in Figure 2 highlights why Edge Recombination excels in real-world stochastic transit environments. Because urban networks impose severe latent risk structures along heavy-traffic corridors, an algorithm must prioritize matching parental edges. Position-based methods break these connections arbitrarily, resulting in low success rates and delayed convergence times.

Furthermore, tracking population diversity (Figure 3) reveals that ER avoids premature saturation, maintaining genetic diversity to discover superior global solutions. As shown in Figure 4, while exact solvers scale poorly ($O(2^n)$) on large networks, our model maintains efficient, linear-logarithmic scaling performance, making it highly viable for large-scale real-world transit deployments.

Unlike classical stochastic programming approaches, recent distributionally robust models emphasize resilience against distributional shifts and model misspecification [17]. The

present study extends these ideas to Hamiltonian cycle optimization under urban transportation uncertainty.

In conclusion, this research proves that marrying Distributionally Robust Optimization with heuristic evolutionary frameworks offers a powerful, mathematically tractable approach to solving combinatorial graph routing problems under severe uncertainty. By replacing brittle nominal expectations with an infinite-dimensional Wasserstein ambiguity set, our formulation provides explicit distribution-free safety guarantees capable of weathering highly volatile non-stationary urban environments. Future research directions will evaluate the scalability of the dualized linear reformulations using decomposed parallel computing clusters.

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